

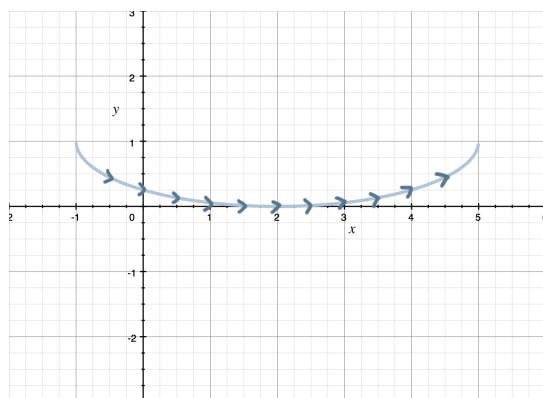


# Precalculus Final Exam Solutions

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## Precalculus Final Exam Answer Key

1. (5 pts) ☐ A ☒ ☐ C ☐ D ☐ E
2. (5 pts) ☒ ☐ B ☐ C ☐ D ☐ E
3. (5 pts) ☐ A ☐ B ☐ C ☐ D ☒ E
4. (5 pts) ☐ A ☐ B ☒ ☐ D ☐ E
5. (5 pts) ☐ A ☐ B ☒ ☐ D ☐ E
6. (5 pts) ☐ A ☐ B ☐ C ☐ D ☒ E
7. (5 pts) ☐ A ☐ B ☐ C ☒ ☐ E
8. (5 pts) ☐ A ☒ ☐ C ☐ D ☐ E



9. (15 pts)

10. (15 pts)  $(\sqrt{14}, 0)$  and  $(-\sqrt{14}, 0)$

11. (15 pts)  $-64$

12. (15 pts)  $z_1 = 2 \left( \cos \frac{9\pi}{32} + i \sin \frac{9\pi}{32} \right)$

$z_3 = 2 \left( \cos \frac{41\pi}{32} + i \sin \frac{41\pi}{32} \right)$

$z_2 = 2 \left( \cos \frac{25\pi}{32} + i \sin \frac{25\pi}{32} \right)$

$z_4 = 2 \left( \cos \frac{57\pi}{32} + i \sin \frac{57\pi}{32} \right)$



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1. B. We're solving for angle  $\alpha$ , so first take the inverse tangent of both sides.

$$\tan \alpha = 1$$

$$\alpha = \tan^{-1} 1$$

Since

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$\tan \theta = 1$  occurs when sine and cosine are equal. This happens at  $45^\circ$  because

$$\sin 45^\circ = \frac{\sqrt{2}}{2} \text{ and } \cos 45^\circ = \frac{\sqrt{2}}{2}$$

Therefore,

$$\alpha = 45^\circ$$

2. A. We've been asked to convert the polar coordinates  $(r, \theta)$  to rectangular coordinates  $(x, y)$ . Remember that  $x = r \cos \theta$  and  $y = r \sin \theta$ . In this problem,  $r = 2$  and  $\theta = \pi/3$ . So the value of  $x$  is

$$x = 2 \cos \frac{\pi}{3}$$



$$x = 2 \left( \frac{1}{2} \right)$$

$$x = 1$$

and the value of  $y$  is

$$y = 2 \sin \frac{\pi}{3}$$

$$y = 2 \left( \frac{\sqrt{3}}{2} \right)$$

$$y = \sqrt{3}$$

The rectangular coordinates are  $(1, \sqrt{3})$ .

3. E. Solve the conversion equation  $x = r \cos \theta$  for  $\cos \theta$ .

$$\cos \theta = \frac{x}{r}$$

Now we can substitute into the equation.

$$r = 4 \cos \theta$$

$$r = 4 \frac{x}{r}$$

$$r^2 = 4x$$

Substitute  $r^2$  using the conversion equation  $r^2 = x^2 + y^2$ .



$$x^2 + y^2 = 4x$$

This is the equation of a circle. To find the standard form of the equation, we'll need to move all terms to one side and complete the square.

$$x^2 - 4x + y^2 = 0$$

Complete the square by dividing the coefficient of  $x$  by 2 and squaring the result.

$$\frac{-4}{2} = -2$$

$$(-2)^2 = 4$$

Add 4 to both sides of the equation.

$$x^2 - 4x + 4 + y^2 = 4$$

Factor  $x^2 - 4x + 4$ .

$$(x - 2)^2 + y^2 = 4$$

This is the equation of the circle whose center has rectangular coordinates  $(2,0)$  and whose radius is 2.

4. C. The polar curve is a rose, and since the petals lie on the  $x$ - and  $y$ -axes, the equation is in the form  $r = a \cos n\theta$ . The radius of the petals is 6, so  $a = 6$ . Since there are 8 petals,  $2n = 8$  and  $n = 4$ . Therefore, the polar equation is  $r = 6 \cos 4\theta$ .



5. C. First, find the equation of the parabola in the standard form  $x - h = a(y - k)^2$ .

$$y^2 + 4y - x + 1 = 0$$

$$x - 1 = y^2 + 4y$$

$$x - 1 + 4 = y^2 + 4y + 4$$

$$x + 3 = (y + 2)^2$$

The focus is at  $(h + (1/4)a, k)$ .

$$\left(-3 + \left(\frac{1}{4}\right)(1), -2\right)$$

$$\left(\frac{-12}{4} + \frac{1}{4}, -2\right)$$

$$\left(\frac{-11}{4}, -2\right)$$

6. E. Start by finding the standard form of the ellipse.

$$x^2 - 2x + 2y^2 + 12y - 17 = 0$$

$$x^2 - 2x + 2(y^2 + 6y) = 17$$

$$x^2 - 2x + 1 + 2(y^2 + 6y + 9) = 17 + 1 + 18$$



$$(x - 1)^2 + 2(y + 3)^2 = 36$$

$$\frac{(x - 1)^2}{36} + \frac{(y + 3)^2}{18} = 1$$

Now we see that  $a = \sqrt{36} = 6$  and  $b = \sqrt{18}$ . To find eccentricity, we need to find  $c/a$ . Use the formula  $b^2 + c^2 = a^2$  to find  $c$ .

$$(\sqrt{18})^2 + c^2 = 6^2$$

$$18 + c^2 = 36$$

$$c^2 = 18$$

$$c = \sqrt{18}$$

Therefore,

$$\frac{c}{a} = \frac{\sqrt{18}}{6} \approx 0.7071$$

7. D. Rewrite the equation so that all the  $i$  terms are written as multiples of  $i^2$  and  $i$ .

$$5i^2 - 2i^3 + i^4 + 4i$$

$$5(i^2(-2(i^2(i + (i^2)(i^2)) + 4i$$

Substitute  $-1$  for  $i^2$ .

$$5(-1) - 2(-1)i + (-1)(-1) + 4i$$



$$-5 + 2i + 1 + 4i$$

$$-4 + 6i$$

8. B. To multiply complex numbers, multiply the outside numbers and add the angles.

$$\left[ 2\sqrt{3} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \right] \left[ \frac{4}{\sqrt{3}} \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \right]$$

$$2\sqrt{3} \cdot \frac{4}{\sqrt{3}} \left[ \cos \left( \frac{3\pi}{4} + \frac{\pi}{3} \right) + i \sin \left( \frac{3\pi}{4} + \frac{\pi}{3} \right) \right]$$

$$8 \left[ \cos \left( \frac{9\pi}{12} + \frac{4\pi}{12} \right) + i \sin \left( \frac{9\pi}{12} + \frac{4\pi}{12} \right) \right]$$

$$8 \left( \cos \frac{13\pi}{12} + i \sin \frac{13\pi}{12} \right)$$

9. Start by solving the equations for  $\sin t$  and  $\cos t$ . We get

$$x = 2 + 3 \sin t$$

$$\sin t = \frac{x - 2}{3}$$

and



$$y = 1 - \cos t$$

$$\cos t = 1 - y$$

Use the basic Pythagorean identity  $\sin^2 t + \cos^2 t = 1$  and substitute the results above.

$$\left(\frac{x-2}{3}\right)^2 + (1-y)^2 = 1$$

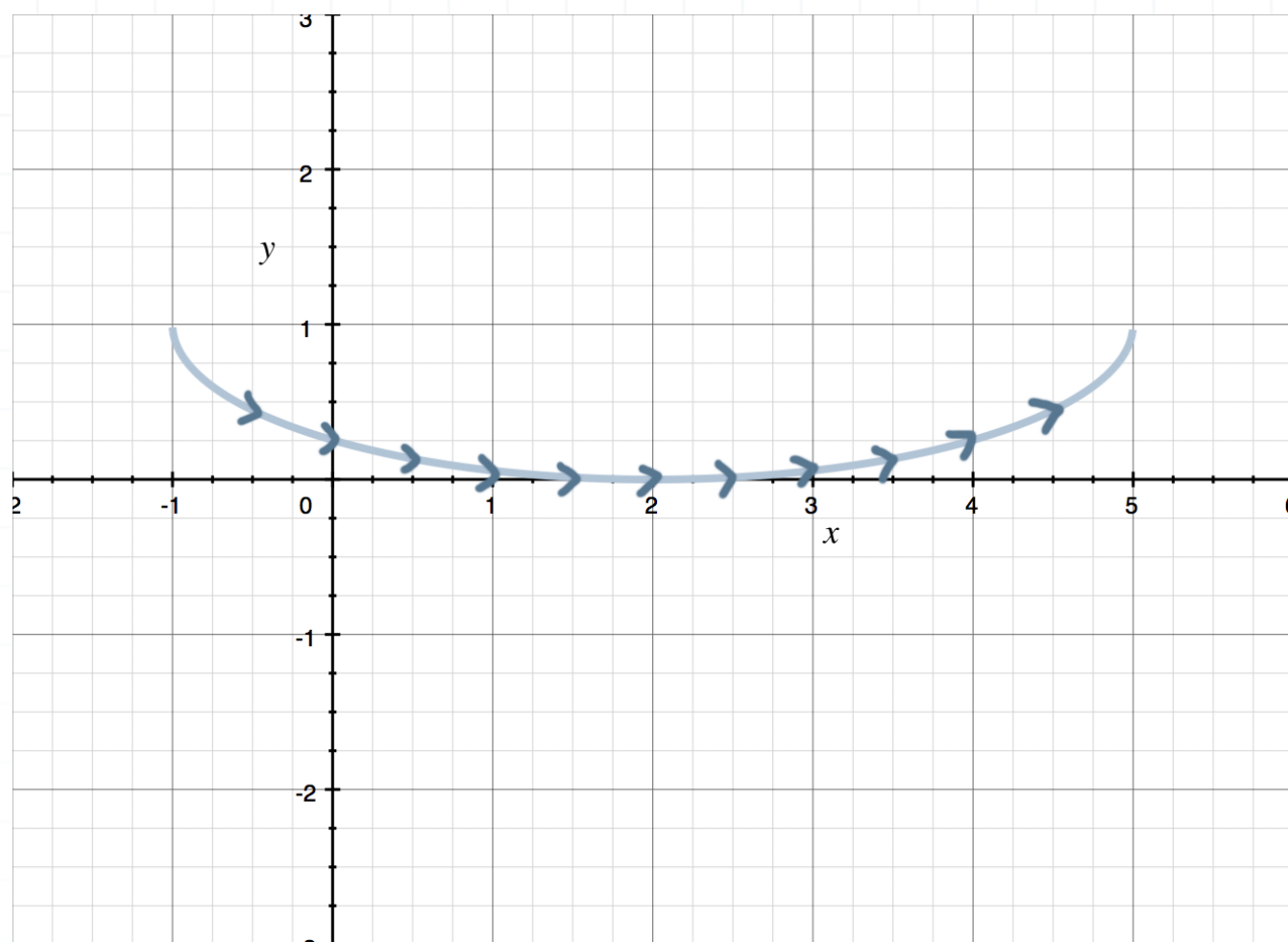
$$\frac{(x-2)^2}{9} + (1-y)^2 = 1$$

This is the equation of an ellipse, and we now know the shape of the graph. Let's make a table of values to help us sketch the graph.

| $t$              | $x = 2 + 3 \sin t$                                                              | $y = 1 - \cos t$                                                            |
|------------------|---------------------------------------------------------------------------------|-----------------------------------------------------------------------------|
| $-\frac{\pi}{2}$ | $2 + 3 \sin\left(-\frac{\pi}{2}\right) = 2 + 3(-1) = -1$                        | $1 - \cos\left(-\frac{\pi}{2}\right) = 1 - 0 = 1$                           |
| $-\frac{\pi}{4}$ | $2 + 3 \sin\left(-\frac{\pi}{4}\right) = 2 - \frac{3\sqrt{2}}{2} \approx -0.21$ | $1 - \cos\left(-\frac{\pi}{4}\right) = 1 - \frac{\sqrt{2}}{2} \approx 0.29$ |
| 0                | $2 + 3 \sin 0 = 2 + 3(0) = 2$                                                   | $1 - \cos 0 = 1 - 1 = 0$                                                    |
| $\frac{\pi}{4}$  | $2 + 3 \sin \frac{\pi}{4} = 2 + \frac{3\sqrt{2}}{2} \approx 4.21$               | $1 - \cos \frac{\pi}{4} = 1 - \frac{\sqrt{2}}{2} \approx 0.29$              |
| $\frac{\pi}{2}$  | $2 + 3 \sin \frac{\pi}{2} = 2 + 3(1) \approx 5$                                 | $1 - \cos \frac{\pi}{2} = 1 - 0 = 1$                                        |



Plot the points in order, drawing arrows in the same direction as you plot the points (counter-clockwise).



10. Square each equation and simplify. For  $x$  we get

$$x = 7t + \frac{1}{2t}$$

$$x^2 = \left(7t + \frac{1}{2t}\right)^2$$

$$x^2 = 49t + \frac{1}{4t^2} + 7$$

and for  $y$  we get



$$y = 7t - \frac{1}{2t}$$

$$y^2 = \left(7t - \frac{1}{2t}\right)^2$$

$$y^2 = 49t + \frac{1}{4t^2} - 7$$

To find the vertices, subtract  $y^2$  from  $x^2$ .

$$x^2 - y^2 = 49t + \frac{1}{4t^2} + 7 - \left(49t + \frac{1}{4t^2} - 7\right)$$

$$x^2 - y^2 = 49t + \frac{1}{4t^2} + 7 - 49t - \frac{1}{4t^2} + 7$$

$$x^2 - y^2 = 14$$

Therefore, the vertices of the hyperbola are at

$$\left(\sqrt{14}, 0\right) \text{ and } \left(-\sqrt{14}, 0\right)$$

11. To find the complex number raised to the power of 4, raise the outside number to the power of 4 and multiply the angle by 4.

$$\left(2\sqrt{2}\right)^4 \left[ \cos \left(4 \cdot \frac{5\pi}{4}\right) + i \sin \left(4 \cdot \frac{5\pi}{4}\right) \right]$$

$$16 \cdot 4 (\cos 5\pi + i \sin 5\pi)$$



$$64 \cos 5\pi + 64i \sin 5\pi$$

$$64(-1) + 64(0)$$

$$-64$$

12. There will be 4 roots since we're taking the fourth root of a complex number. Find the first root by taking the fourth root of the outside number and dividing the angle by 4.

$$\sqrt[4]{16} \left[ \cos \left( \frac{1}{4} \cdot \frac{9\pi}{8} \right) + i \sin \left( \frac{1}{4} \cdot \frac{9\pi}{8} \right) \right]$$

$$z_1 = 2 \left( \cos \frac{9\pi}{32} + i \sin \frac{9\pi}{32} \right)$$

To find the other three roots, divide  $2\pi$  by 4 (since we're taking the fourth root) and add the result to the previous angle.

$$\frac{2\pi}{4} = \frac{\pi}{2} = \frac{16\pi}{32}$$

Then the second root is

$$z_2 = 2 \left[ \cos \left( \frac{9\pi}{32} + \frac{16\pi}{32} \right) + i \sin \left( \frac{9\pi}{32} + \frac{16\pi}{32} \right) \right]$$

$$z_2 = 2 \left( \cos \frac{25\pi}{32} + i \sin \frac{25\pi}{32} \right)$$



the third root is

$$z_3 = 2 \left[ \cos \left( \frac{25\pi}{32} + \frac{16\pi}{32} \right) + i \sin \left( \frac{25\pi}{32} + \frac{16\pi}{32} \right) \right]$$

$$z_3 = 2 \left( \cos \frac{41\pi}{32} + i \sin \frac{41\pi}{32} \right)$$

and the fourth root is

$$z_4 = 2 \left[ \cos \left( \frac{41\pi}{32} + \frac{16\pi}{32} \right) + i \sin \left( \frac{41\pi}{32} + \frac{16\pi}{32} \right) \right]$$

$$z_4 = 2 \left( \cos \frac{57\pi}{32} + i \sin \frac{57\pi}{32} \right)$$



